S Parameter Reference Nodes

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1.0 Summary

This document addresses the question
"How are the port reference nodes for an S parameter matrix related?"

In other words, do all the ports for an S parameter matrix share a common ground reference? If not, how does one account for differences in potential between the ground reference for one port and the next? What if two ports of an S parameter matrix share a common ground because they're the two sides of a differential pair? Does this affect the relationship to any other ports?

This document answers this question with increasing levels of detail. The shortest answer to this question is
The port reference nodes for an S parameter matrix don't have to be related to each other in any way whatsoever.

The next longer answer is
Ports of an S parameter matrix may share a common reference node, especially if the ports represent different propagation modes for a single transmission structure; however that would not change the definition or function of the reference node for any other port.

The next, even longer answer is
S parameters are defined by the measurement system and not the device under test. In particular, S parameters are defined in terms of incident and reflected waves in transmission structures which are part of the measurement system and not the device under test. For each port, the reference node is that of the transmission structure used to make the measurement, which may have little or nothing to do with the nature or construction of the device under test. Although S parameters were originally defined for measurements using single mode transmission structures, that definition can be extended to cover multi-mode transmission structures as well.

The remainder of this document address the following points and topics:

- The textbook definition of S parameters demonstrates that the ports and their reference nodes are defined by the measurement system and not the device under test.
- The way S parameters are almost always measured enforces the independence of reference nodes in a way that is entirely consistent with the textbook definition.
- As an examples, analysis of a common ground impedance demonstrates how the relationship between reference nodes in a device under test will be reflected in the S parameter matrix.

- The concept of S parameters can be extended measurements using transmission structures which support multiple propagation modes rather than a single propagation mode. In this case, multiple ports may share a common reference node.

2.0 Textbook Definition

Ramo, Whinnery and Van Duzer [1] describes S parameters using a drawing similar to Figure 1

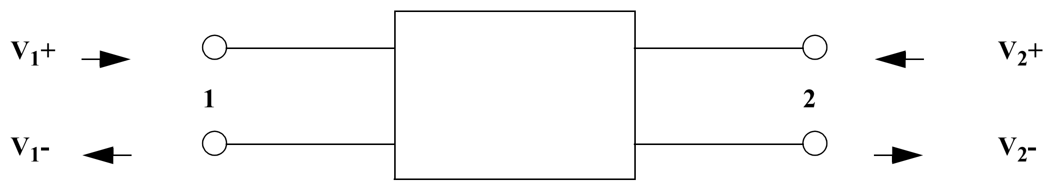

FIGURE 1. Incident and reflected waves at ports of microwave network.

The associated discussion assumes that a transmission structure such as a waveguide or transmission line is connected at port 1 and a second one at port 2. Each such transmission structure supports incident waves V_n+ and reflected waves V_n-. The associated equations are

$$a_n \equiv \frac{V_{n+}}{\sqrt{Z_{0n}}} \qquad b_n \equiv \frac{V_{n-}}{\sqrt{Z_{0n}}} \quad \text{(EQ 1)}$$

$$V_n = V_{n+} + V_{n-} = \sqrt{Z_{0n}}(a_n + b_n) \quad \text{(EQ 2)}$$

$$I_n = \frac{1}{Z_{0n}}(V_{n+} - V_{n-}) = \frac{1}{\sqrt{Z_{0n}}}(a_n - b_n) \quad \text{(EQ 3)}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \text{(EQ 4)}$$

Notice that the above equations are explicitly based on the assumption that the measurement is being made using single mode transmission structures, but no assumption whatsoever is made about the device under test.

If the measurement is made using transmission lines, then the reference node for each port is the ground of the associated tranmission line. There is no assumption that the grounds of the respective transmission lines are related in any way.

3.0 Measurement Systems

While the first S parameter measurements were made using slotted waveguides, almost all modern S parameter measurements are made using coaxial cables, possibly extended by planar transmission lines. The point where each measurement transmission line contacts the device under test is a port of the S parameter matrix, and the ground conductor of the transmission line at that point is the reference node for that port.

Note that there is nothing to guarantee that the grounds of the different measurement transmission lines are somehow tied together at or near the device under test. For example, people do make S parameter measurements between antennas in which there is a free space propagation path in the middle of the device under test.

If there are ground potential differences between the different measurement ports, as there can very well be, those potential differences will generate currents that flow along the ***outside*** of the measurement cables and get shorted out at the faceplate of the measurement test set. At the frequency of measurement, these currents do not get to the inside of the measurement cables, and therefore do not affect the measurement. Note that this principle does not apply at frequencies for which the cable shield is less than about a skin depth thick; however, in principle one would then use cables with thicker shields.

4.0 Example

Suppose one wants the S parameter matrix for the structure shown in Figure 2.

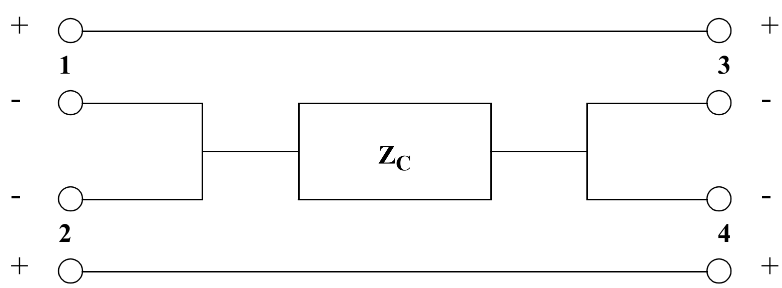

FIGURE 2. Common mode ground impedance

The S parameters for this structure are

$$S_{11} = S_{22} = S_{33} = S_{44} = \frac{Z_C Z_0}{Z_C Z_0 + Z_0^2} \quad \text{(EQ 5)}$$

$$S_{13} = S_{31} = S_{24} = S_{42} = \frac{\frac{3}{4} Z_C Z_0 + Z_0^2}{Z_C Z_0 + Z_0^2} \quad \text{(EQ 6)}$$

$$S_{12} = S_{14} = S_{21} = S_{23} = S_{32} = S_{34} = S_{41} = S_{43} = -\frac{Z_C Z_0}{Z_C Z_0 + Z_0^2} \qquad \text{(EQ 7)}$$

5.0 Multi-mode Measurements

The concept of S parameters can be extended to measurements made with transmission structures which support more than one propagation mode. One popular example is twinax cable, in which there are two signal conductors surrounded by a single shield conductor. This type of cable supports two propagation modes: even mode (sometimes called common mode) and odd mode (sometimes called differential mode). Another example would be a waveguide which is large enough to propagate more than one mode at the frequency of measurement. An antenna measurement would be an extreme example in that each combination of azimuth and elevation has a separate propagation mode associated with it.

When there are multiple modes in a transmission structure used for measurement, each mode in that structure must be considered to be a separate port. That is, there are incident and reflected waves for each propagation mode. Furthermore, the device under test can cause consersions between modes, resulting in reflected waves with different modes than those of the incident waves. In other words, there can and usually will be coupling between modes. As long as this principle is observed, then equations 1-4 are just as applicable to measurements made with a multi-mode transmission structure as they are to the traditional single mode structures.

When a multi-mode transmission structure is used for measurement, it just so happens that the ports defined for the different modes of that structure share a common reference node. For example, the even and odd modes in a twinax cable share the same shield as a reference node. This does not, however, imply that there should be any relationship between that reference node and the reference nodes for any other single or multi-mode transmission structure used in the measurement, nor does it imply that the S parameter matrix contains any information about the relationship between reference nodes, nor does it imply that the relationship between reference nodes is at all relevant to the definition of the S parameters.

6.0 References

[1] Ramo, Whinnery and Van Duzer, *Fields and Waves in Communication Electronics*, Third Edition, John Wiley and Sons, copyright 1994, pg. 539-41.